

IYPT 2000

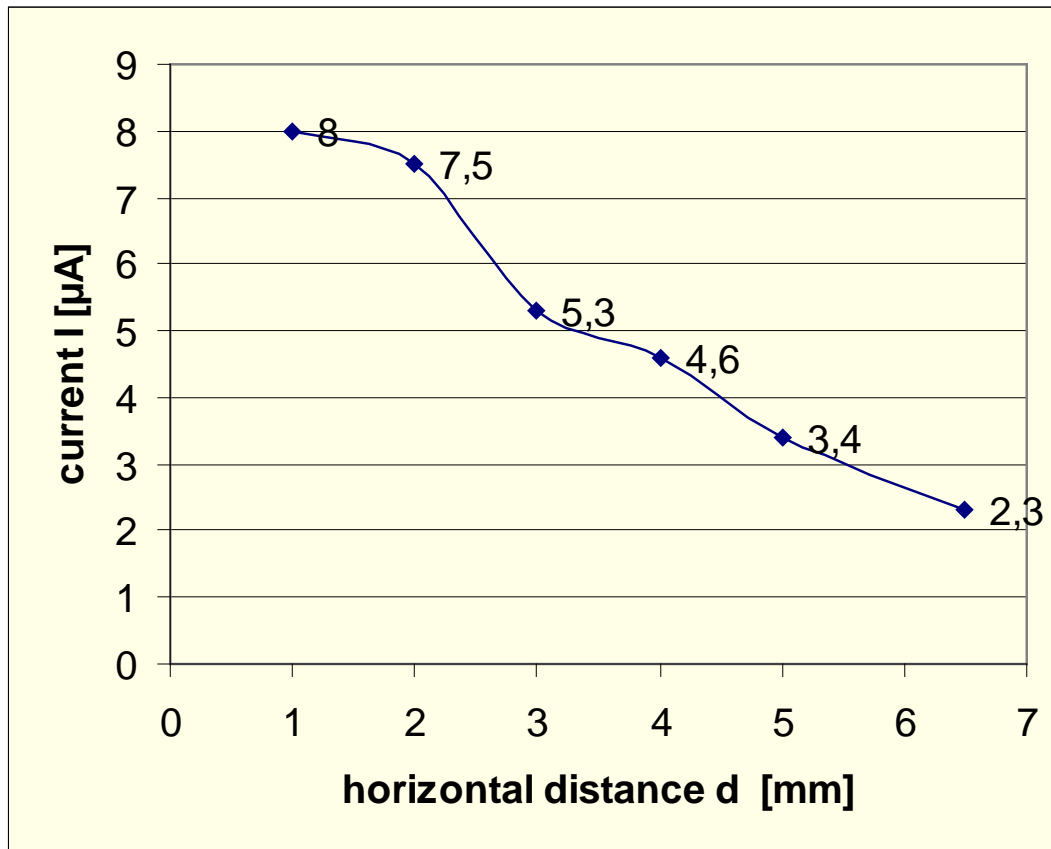
Problem No. 3

PLASMA

Team Austria

Investigate the electrical conductivity of the flame of a candle. Examine the influence of relevant parameters, in particular, the shape and polarity of the electrodes. The experiments should be carried out with a voltage not exceeding 150 V.

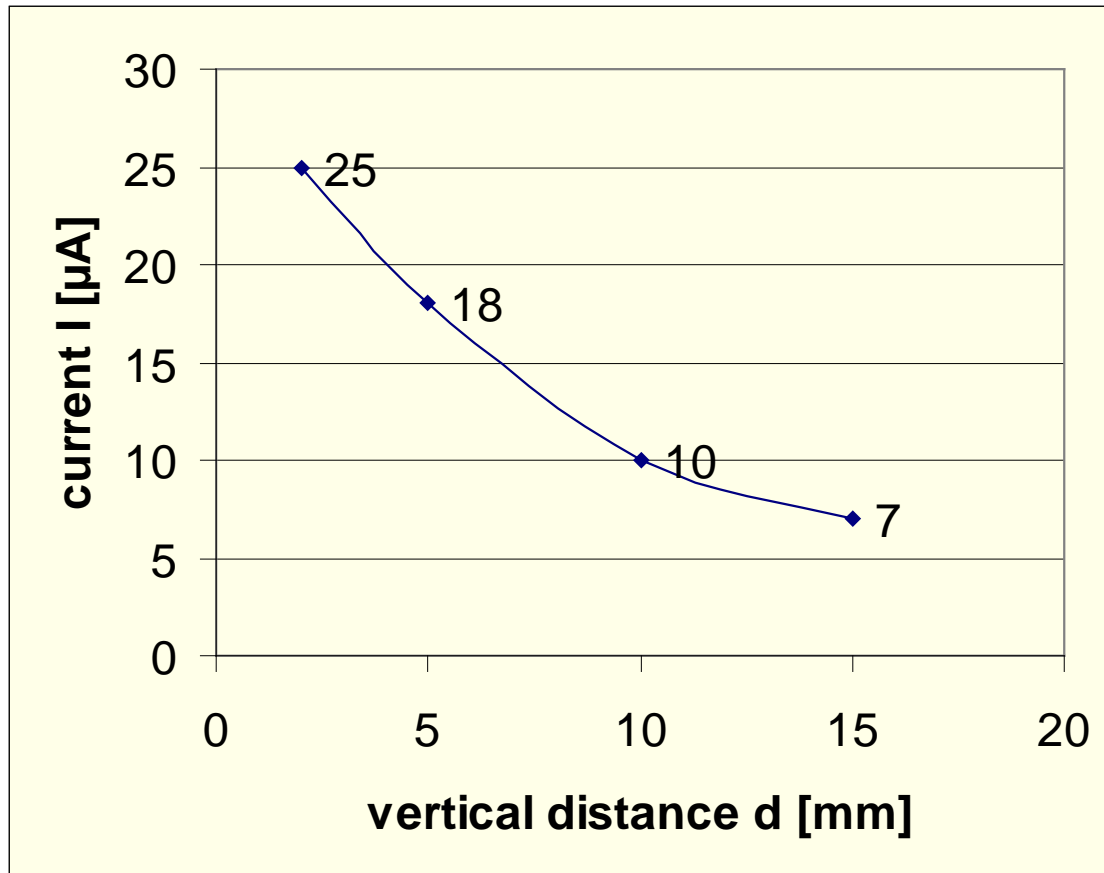
Measurements



experimental data:

- sharp electrodes
- on the same level in the flame
- voltage: 150 V, DC

Measurements



experimental data:

- sharp electrodes
- above each other
- Intersection: about 4 mm
- voltage: 150 V, DC

Conclusions from the measurements

The problem is *not* completely **solveable!**

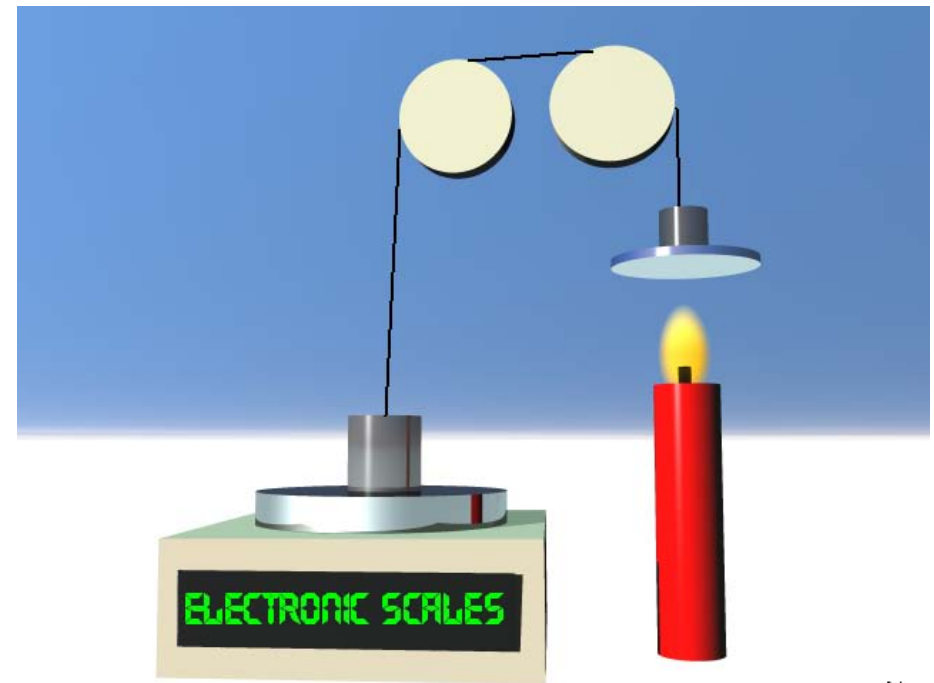
Only the influence of various parameters can be examined:

- electric field (shape and arrangement of the electrodes)
- temperature
- gas flow
- composition of the gas

Gas Flow - Experiment

left mass: 2g; right mass: 1g; connected with a circular **Al** – plate
(in the flame: **Fe** – plate); Temperature measurement with Pt-100
or thermocouple

The streaming force F_{str} is an upward force on the right mass and produces an additional downward force on the electronic scales. This causes the scales to show a higher weight, so F_{str} can be calculated using this “mass difference” Δm .



Gas Flow - Calculations

$$F_{\text{Str}} = \frac{c_w \cdot A \cdot \rho \cdot v^2}{2} \Rightarrow v = \sqrt{\frac{2 \cdot F_{\text{Str}}}{c_w \cdot A \cdot \rho}}$$

$C_w = 1,1$ (for circular plates)
 $A = r^2 \cdot \pi$ (area of the circular plate)
 F_{Str} ... has been measured
 ρ ... density of air: calculated (see following transparent)

example of calculation: distance from the flame: 50 cm

$$\Delta m = 5 \cdot 10^{-6} [\text{kg}]$$

$$F_{\text{Str}} = 4,905 \cdot 10^{-5} [\text{N}]$$

$$\vartheta = 22,5^\circ\text{C} \Rightarrow \rho = 1,195 \left[\frac{\text{kg}}{\text{m}^3} \right]$$

$$v = \sqrt{\frac{2 \cdot F_{\text{Str}}}{c_w \cdot A \cdot \rho}} = \sqrt{\frac{2 \cdot 4,905 \cdot 10^{-5}}{1,1 \cdot 1,256 \cdot 10^{-3} \cdot 1,195}}$$

$$v \approx 0,24 \left[\frac{\text{m}}{\text{s}} \right]$$

Gas Flow - Calculations

Calculation of air density: ideal gas equation

$$p \cdot V = n \cdot R \cdot T$$

p pressure [Pa]

$$p \cdot V = \frac{m}{M} \cdot R \cdot T$$

V volume [m³]

$$p \cdot M = \frac{m}{V} \cdot R \cdot T$$

n number of moles

$$p \cdot M = \rho \cdot R \cdot T$$

R gas constant (= 8,31) [J / (K . mol)]

T temperature [K]

M molecular weight

ρ density [kg/m³]

$$\rho = \frac{p \cdot M}{R \cdot T}$$

calculation of the molecular weight M:

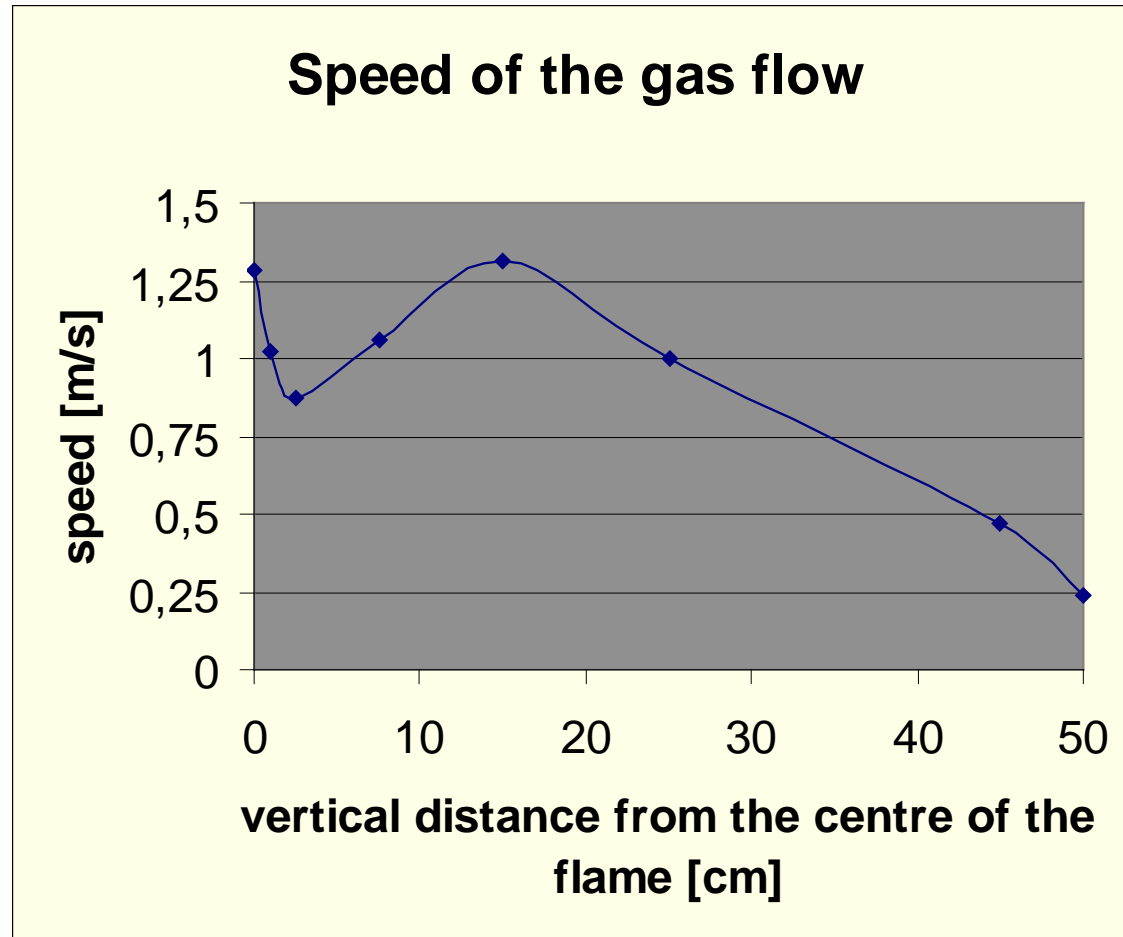
air at 0°C, 1 bar: $\rho = 1,293$ [kg/m³]

$$M = \frac{\rho \cdot R \cdot T_0}{p_0} = \frac{1,293 \cdot 8,31 \cdot 273,15}{10^5}$$

$$M = 0,02935[\text{kg}]$$

(ρ [kg/m³] → M [kg])

Gas Flow - Results



Theory about plasma

Plasma in thermic equilibrium: 3 distributions

- Boltzmann distribution
- Maxwell distribution
- Saha distribution

For a plasma in thermic equilibrium, all 3 distributions have to be fulfilled.

Maxwell distribution

$$N(v) = 4\pi N \left(\frac{m_M}{2\pi k T} \right)^{\frac{3}{2}} v^2 \cdot e^{-\frac{m_M v^2}{2kT}}$$

N ...number of molecules

k ...Boltzmann constant ($\approx 1,38 \cdot 10^{-23}$ [J/K])

m_M ...mass of one molecule

T ...temperature [K]

Ionisation through hits

calculation of the required minimum speed for ionisation:

$$E_i = \frac{m_M \cdot v_{\min}^2}{2}$$

Maxwell distribution to calculate the fraction of molecules which are faster than v_{\min} :

$$x = 4\pi \left(\frac{m_M}{2\pi k T} \right)^{\frac{3}{2}} \int_{v_{\min}}^{\infty} v^2 \cdot e^{-\frac{m_M v^2}{2kT}} dv$$

N...number of molecules

k...Boltzmann constant ($\approx 1,38 \cdot 10^{-23}$ [J/K])

m_M ...mass of one molecule

T...temperature [K]

results: e.g. for O_2 : $x \approx 3 \cdot 10^{-64}$

for N_2 : $x \approx 9 \cdot 10^{-91}$

Saha distribution

The Saha equation indicates the ionisation balance between the electrons and the ions versus neutral particles:

$$S = \frac{N_e^- \cdot N_i^+}{N} = \frac{(2 \cdot \pi \cdot m_e)^{\frac{3}{2}} \cdot (k \cdot T)^{\frac{5}{2}} \cdot e^{-\frac{E_i}{k \cdot T}}}{h^3}$$

m_emass of the electron

hPlanck constant

kBoltzmann constant

Ttemperature

E_iionisation energy

like chemical equilibrium:



Chemical equation for the ionisation of an element X.

$$K = \frac{[X^+] \cdot [e^-]}{[X]} = f(T, E_i)$$

Influence of the electric field

The electric field causes electrons to drift through the plasma with a current density j :

$$j = N_e \cdot e \cdot v_d$$

moreover:

$$j = \sigma \cdot E$$

$$v_d = \frac{e}{m_e} \cdot \tau \cdot E$$

$$v_d = \mu \cdot E$$

$$\lambda_e = v_e \cdot \tau$$

$$v_e = \sqrt{\frac{8 \cdot k \cdot T}{\pi \cdot m_e}}$$

N_e ...number of electrones

e ...elementary charge

v_d ...drifting speed

E ...electric field strength

m_e ...mass of the electron

τaverage time between 2 hits

σ ...conductivity

μ ...mobility of the electrons

λ_efree distance between 2 hits

v_eaverage speed of the electrons (Maxwell distribution)

kBoltzmann constant

Ttemperature

calculation

using the shown formulas:

$$j = N_e \cdot e \cdot \mu_e \cdot E = \sigma \cdot E$$

$$\Rightarrow \sigma = \frac{1}{2} \sqrt{\frac{\pi}{2}} \cdot \frac{e^2}{\sqrt{m_e \cdot k \cdot T}} \cdot N_e \cdot \lambda_e$$

$$\Rightarrow \sigma \sim \frac{1}{\sqrt{m}}$$

Taking an element with an average mass from the periodic system of the elements we can show that the share of ions of the current is $< 0,4 \%$.

So we can restrict ourselves to the consideration of drifting electrons only.

Richardson equation

$$j = A \cdot T^2 \cdot e^{-\frac{W_A}{k \cdot T}}$$

$$A = \frac{4\pi \cdot m \cdot k \cdot T}{h^3}$$

W_A ...work function

kBoltzmann constant

TTemperature

hPlanck constant

mrest mass of the electrone

The Richardson equation shows only the influence of the temperature and is *not considering the point effect!*

Field influence

field emission: Schottky-effect (point effect)

Fowler-Nordheim equation:

$$E = \frac{F}{Q} = \frac{F}{e_c}$$

$$j \sim E^2 \cdot e^{-\frac{E_0}{E}}$$

E....electric field strength

F....force

Q....charge

e_c ...elementary charge

j.....current density